1st Junior Penchick Online Mathematical Olympiad

Qualifying Stage, Paper C

(Ages 14 and below)

9 May 2026

INSTRUCTIONS AND INFORMATION

1. **There are 20 questions to be answered in 90 minutes, split into two parts.** The highest possible score for this contest is 48 points.
   - *Part I (multiple-choice):* Each question has four choices (A, B, C, and D), and only one of these choices is correct. Submit the best answer to each question. Two points are given for a correct answer, for a total of 24 points for this section.
   - *Part II (short answer):* Submit the best answer to each question. Three points are given for a correct answer, for a total of 24 points for this section. For this paper, all answers are **integers** from 0 to 999 inclusive.
2. **Always exhibit academic honesty during the contest.** Do not use calculators, protractors, graph paper, search engines, artificial intelligence, and other external applications, resources, or help while answering the contest. Rulers, compasses, and blank or lined paper are allowed. Cheating in any form is strictly prohibited and will be acted upon based on the violation system.
3. Note that diagrams are **not necessarily drawn to scale**.
4. The top 60% of contestants in this paper will be invited to take the **1st JPOMO Final Stage** on 27 June 2026.

Enjoy the contest!

Part I: Multiple-Choice Questions

Each question has four choices (A, B, C, and D), and only one of these choices is correct. Submit the best answer to each question. Two points are given for a correct answer, for a total of 24 points for this section.

1. The Penchick Cinema stores the number of spectators for each movie. There are 11 records, which are:

$$23,\ 43,\ 24,\ 51,\ 61,\ 38,\ 21,\ 47,\ 38,\ 28,\ 55$$

Which of the following is the largest?

(a) The median of these results
(b) The average of these results
(c) The mode of these results
(d) The range of these results

2. Let $n = \dfrac{10!}{k}$, where n and k are natural numbers and k is a perfect square. What is the largest possible value of k?

(a) 2^2 (b) 80^2 (c) 360^2 (d) 720^2

3. Renren is making a wacky drink in several stages. First, he prepares an orange squash by mixing water and orange juice in the ratio 3 : 1. Next, he prepares a blackcurrant squash by mixing water and blackcurrant juice in the ratio 5 : 1. He then combines the two squashes to make a "blackorange" squash, using 2 L of blackcurrant squash and 3 L of orange squash. Finally, for every 5 L of blackorange squash, he adds 500 mL of water and 250 mL of fish oil. Find the final ratio of *fish oil : water : juice.*

(a) 3 : 53 : 13 (b) 3 : 49 : 11 (c) 3 : 57 : 13 (d) 3 : 43 : 21

4. What is the largest possible value of the greatest common divisor for pairs of numbers in the form $(27x + 12,\ 30x + 15)$, where x is an integer?

(a) 1 (b) 3 (c) 5 (d) 15

5. A laboratory stores penchickium in cubic tanks. Each tank contains 1.2 m^3 of penchickium. Penchickium has a density of 1.5 g/cm^3. Penchickium is to be transferred into small bottles, each of volume 375 mL. How many bottles are needed to transfer all the penchickium from one tank?

(a) 2,400 (b) 3,000 (c) 3,200 (d) 4,000

6. Let ABC be a right triangle with $\angle C = 90^\circ$, $AC = x$ and $BC = x\sqrt{3}$. Consider a point D on segment AB. How many points can we place D in such that it forms a $30-60-90$ triangle $\triangle ADC$ with $\angle ADC = 60^\circ$?

(a) 0 (b) 1 (c) 2 (d) ∞

7. Chubby wants to buy two different games! Sadly, he forgot to wishlist them, so he turns to you in order to figure out the possible costs for the games to find them! He knows that one of them costs x dollars and the other costs $x\%$ more. Also, Chubby notices that the more expensive game costs $2x-9$ dollars. What is the sum of all possible values for the cost of both games?

(a) 21 (b) 100 (c) 261 (d) 282

8. Let $f(x) = 60x^2 + bx + c$ for some real numbers b and c, and let $\triangle ABC$ be a triangle. If $\angle A = f(-1)^\circ$, $\angle B = f(0)^\circ$, and $\angle C = f(1)^\circ$, what is the constant term of $f(x)$?

(a) 20 (b) 40 (c) 60 (d) 80

9. Find the sum of the finite series

$$\sum_{n=2}^{2026} \frac{2}{n^2-1} = \frac{2}{2^2-1} + \frac{2}{3^2-1} + \frac{2}{4^2-1} \cdots + \frac{2}{2026^2-1}.$$

(a) $2 - \dfrac{1}{2027}$ (b) $\dfrac{3041}{2028} - \dfrac{1}{2027}$ (c) $\dfrac{3}{2} - \dfrac{1}{2026}$ (d) $\dfrac{7}{4} - \dfrac{1}{2028}$

10. Seen below is the Penchick's Official Anthem:

> *"Squawk chirp squawk, noot chirp squawk,*
> *Honk squawk honk, chirp squawk honk, Squawk squawk honk, chirp chirp squawk*
> *Noot noot squawk, squawk honk chirp."*

Renren sings this anthem many times, repeating it again after completing. What is the 2026th word he says?

(a) Squawk (b) Chirp (c) Honk (d) Noot

11. A referee must select a group of people from among his 6 boys and 8 girls. The only requirements are that the difference between the number of boys and girls must be a multiple of 4, and the group must have at least one person. Let N be the number of different groups that could be selected. What is N?

(a) 3,092 (b) 4,095 (c) 5,098 (d) 6,000

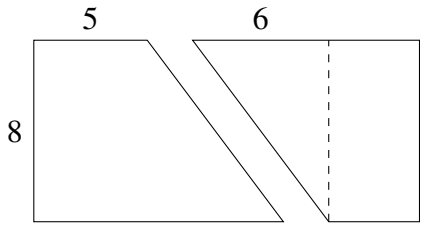
12. Suppose that 7 cards labeled 1 to 7 are placed randomly into 7 boxes also labeled 1 to 7, with exactly one card in each box. What is the probability that no card is placed into a box with the same label as the card?

(a) $\frac{3}{7}$ (b) $\frac{103}{280}$ (c) $\frac{143}{360}$ (d) $\frac{1}{3}$

Part II: Short-Answer Questions

Submit the best answer to each question. Three points are given for a correct answer, for a total of 24 points for this section.
For this paper, all answers are **integers** from 0 to 999 inclusive.

1. A chocolate bar is in the shape of a cuboid with dimension $8 \times 10 \times 15$. Chubby and Pelican split the chocolate by cutting it diagonally, as shown in the figure below. What is the positive difference between the volume of the two pieces?



2. What is the largest integer value of x that satsifies the inequality

$$\frac{1}{x}+\frac{1}{2x}+\frac{1}{2}+\frac{1}{3}+\frac{1}{5}-\frac{1}{15}>1?$$

3. Penrick likes being unfair, and so he flips an unfair coin three times that gives only heads and tails, landing heads with a probability of $\frac{2}{3}$ each time. He also flips another coin thrice that does different things depending on what happened at the last flip (This is called a recursive coin.) At first, it lands heads and tails with the same probability of $\frac{1}{2}$, and then in the next two flips it can do one of two things depending on what the last flip was:

- If the last flip was a heads, then it lands heads with probability $\frac{3}{4}$ and tails otherwise,
- If the last flip was a tails, then it lands tails with probability $\frac{3}{4}$ and heads otherwise.

If the positive difference between the probability of getting two heads on the unfair coin and on the recursive coin can be written in the form $\frac{a}{b}$ for relatively prime positive integers a and b, then what is $a+b$?

4. Pelican, Chubby and Renren each list an arithmetic sequence. Pelican's sequence begins with the following:

$$4, 11, 18, \ldots$$

Chubby's sequence begins with the following:

$$13, 28, 43, \ldots$$

and Renren's sequence begins with the following:

$$6, 25, 44, \ldots$$

What is the smallest positive integer that appears in all three sequences?

5. Given an isosceles triangle $\triangle ABC$ with $AC = BC = 10$ and $AB = 12$, let line l_1 be tangent to the circumcircle of $\triangle ABC$ at A. Extend side BC to meet l_1 at point D. Find $AD + CD$.

6. In trapezoid $ABCD$ with $AB \parallel CD$, BD and AC are diagonals with AC being perpendicular to both AB and CD. If $\angle B = 45^\circ$ and $\angle D = 30^\circ$, then $\left(\dfrac{BD}{AC}\right)^2$ can be written in the form $a + b\sqrt{c}$, where a, b, c are positive integers and c is not divisible by any perfect square. What is $100c + 10b + a$?

7. A wooden panel leans against a vertical wall with its base initially touching the wall and the ground. When the panel slips, the top slides down the wall while the base moves away from the wall along the ground.

First, the top of the panel drops 9 centimeters, while the base moves 7 centimeters away from the wall.

Next, the top drops a further 30 centimeters, and the base moves an additional 20 centimeters away from the wall.

Finally, the top drops another 40 centimeters.

To the nearest integer, how much farther (in centimeters) from the wall does the base move during this final 40 centimeter drop?

8. There are 2016 cups arranged on a table. Under 2015 of the cups, Chubby has placed a sweet, and under exactly one cup he has placed a chocolate. Whoever gets the chocolate is the winner. The positions of the items are unknown. Penrick and Renren play a game as follows. Penrick first selects 1 cup uniformly at random, takes the item underneath it, and removes both the cup and the item from the game. Renren then selects 2 cups, takes the items underneath them, and removes them. Penrick then selects 3 cups, and this process continues, with the players alternately removing $1, 2, 3, 4, \ldots$ cups respectively, until all cups are gone. If this probability of Penrick winning can be written as $\dfrac{p}{q}$ in lowest terms, find $p + q$.

(*Note:* $2016 = 1 + 2 + 3 + \cdots + 63$ is a triangular number.)

Answer Key

Part I: Multiple-Choice Questions

1. d

2. d

3. a

4. d

5. c

6. a

7. d

8. a

9. VOID $\left(\frac{1519}{1013} - \frac{1}{2027}\right)$

10. b

11. b

12. b

Part II: Short-Answer Questions

1. 80

2. 44

3. 353

4. 823

5. 50

6. 325

7. 20

8. 95